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Coherent population trapping of Bose-Einstein condensates: Detection of phase diffusion

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Abstract. Two Bose-Einstein condensates in different Zeeman sublevels can be decoupled from driving light fields in coherent population trapping. A condensate pair with a deterministic entanglement and a controllable value of the relative phase may be prepared by selecting the phase difference between the coherent light fields. The rate of the condensate phase diffusion may be determined from the two-photon resonant absorption of radiation.

PACS. 03.75.Fi Phase coherent atomic ensemble (Bose condensation) – 42.50.Gy Effects of atomic coherence on propagation, absorption, and amplification of light – 05.30.Jp Boson systems

In several recent experiments on dilute gas alkali Bose-Einstein condensates (BECs) two and multiple condensate systems were created in different sublevels of the same atom [1–5]. The condensate mixtures have been realized in magneto-optical traps [1,3–5], as well as in an optical dipole trap [2], which uses optical forces to trap atoms. In this paper we study the quantum interference effects in two BECs driven by two light fields. Since one needs a phase reference to observe a phase, binary mixtures of BECs are especially useful in studies of coherence properties. We show that the coherent population trapping (CPT) [6–9] could possibly be used as a method of measuring the diffusion of the relative phase between two BECs. In addition, BEC pairs with deterministic and controllable value of the relative phase may be engineered using CPT.

In the first observation of a binary BEC mixture two overlapping BECs of $|F=1, m=-1\rangle$ and $|F=2, m=2\rangle$ states of ^{87}Rb were produced using sympathetic cooling [1]. A BEC pair of $|1,-1\rangle$ and $|2,1\rangle$ states of ^{87}Rb was created by a two-photon transition [3–5]. In these experiments atoms were trapped magneto-optically. An evaporatively cooled ^{23}Na gas was transferred to an optical dipole trap [2]. BECs were observed in several different hyperfine levels. Unlike magnetic traps, dipole traps can stably trap atoms in arbitrary spin states.

Two hyperfine states $|1,-1\rangle$ and $|2,1\rangle$ of ^{87}Rb in a binary mixture of BECs can be coupled by a two-photon transition [3–5]. A microwave field excites atoms from $|1,-1\rangle$ to an intermediate level $|2,0\rangle$ which is coupled to state $|2,1\rangle$ by a radiofrequency photon. A BEC is first prepared in level $|1,-1\rangle$ and a part of the condensate is then transferred to level $|2,1\rangle$. The relative phase between the two separated halves was determined by interfering the atoms at a later time [5]. The observations of interference fringes supported the previous evidence of first order coherence of BECs [10]. Varying the evolution time of the two BECs before the interference measurement yielded information about the phase dynamics.

Atomic interactions give rise to diffusion of the relative phase between two BECs. The uncertainty in the value of the phase increases until all phase information is lost. The width of the number distribution in the ground state has a dispersive effect on the BEC self-interactions and the relative phase undergoes quantum collapses and revivals [11,12]. Additional sources of phase diffusion are spatial mode fluctuations [13] and finite temperature decoherence due to the interactions between condensate and non-condensate atoms [14,15].

In this paper we study CPT [6–9] in two BECs occupying two different Zeeman sublevels. Two light beams are assumed to drive transitions in an atomic Λ three-level scheme. The BECs are prepared in both electronic ground states. In CPT a coherent superposition of the two ground states at the two-photon resonance is a non-absorbing state. In such a superposition state the BECs are decoupled from the driving electromagnetic fields and the atom number states of the BECs are entangled. This entanglement is deterministic in a sense that the relative phase between the two BECs is controllable. The macroscopic quantum coherence of BECs may be established in CPT with a particular value of the relative phase by choosing the phase difference between the coherent light fields. Phase diffusion, due to the atomic collisions, destroys CPT in the BECs, so that the absorption is no longer completely suppressed. We show that measuring the absorption of light at the two-photon resonance can determine the rate of diffusion of the relative phase between the BECs.

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We have previously proposed a method of measuring the relative phase diffusion between two BECs based on quantum interference effects [16]. In that paper we considered a short-time response of two BECs driven by radiofrequency fields for which it was possible to ignore the radiative linewidths. In contrast, the spontaneous optical transitions are essential for CPT. In the present case the phase diffusion is determined by the steady-state response of the coherent superposition of BECs *established* by the spontaneously scattered light in CPT.

Interaction of light with binary BECs has previously been proposed as a method to detect the condensate phase by the amplification of phase coherent Raman laser beams [17] or by spontaneous Raman scattering [18–20]. As a consequence of the matter wave coherence behaving like an optical two-photon coherence, atoms could even be optically pumped into the BECs [21].

We consider a spatially overlapping pair of BECs with a Λ three-level scheme of two ground states and a common excited state. The BECs occupy two different Zeeman sublevels $|1\rangle = |g, m-2\rangle$ and $|2\rangle = |g, m\rangle$. The state $|1\rangle$ is optically coupled to the electronically excited state $|3\rangle = |e, m-1\rangle$ by the driving field $\mathbf{E}_A$ having a polarization σ_+ and a frequency ω_A. Similarly, the state $|2\rangle$ is coupled to $|3\rangle$ by the driving field $\mathbf{E}_B$ with a polarization σ_- and a frequency ω_B. The detunings are defined by $\delta_{31} \equiv \omega_A - \omega_{31}$ and $\delta_{32} \equiv \omega_B - \omega_{32}$, where the transition frequency between the hyperfine levels $1 \leftrightarrow 3$ $(2 \leftrightarrow 3)$ is ω_{21} (ω_{32}). Both light fields $\mathbf{E}_A$ and $\mathbf{E}_B$ propagate in the positive z direction. To simplify the analysis, we consider light scattering only into the BEC modes. The spontaneous scattering to the BECs is stimulated by a large number of atoms in the condensates. By spontaneous scattering we mean that the emission is not stimulated by light, although it is stimulated by the atoms. The decay into non-condensate center-of-mass states is also stimulated by the Bose-Einstein statistics. However, at very low temperatures this stimulation is much weaker because most of the particles are in the BECs. In addition to the Bose stimulation of spontaneous emission there is unstimulated radiative free-space decay which is always present. With a sufficiently large number of atoms in the two BECs the free-space decay may be ignored.

The quantum collapse due to the BEC self-interactions has an important effect on the phase diffusion [11,12]. If the population of level $|3\rangle$ is small, the collisional interactions are mainly between atoms in levels $|1\rangle$ and $|2\rangle$. The collapse rate dramatically depends on the relative strength of the three scattering lengths describing the interspecies (u_{12}) and intraspecies (u_1 and u_2) scattering of atoms in the two ground states $|1\rangle$ and $|2\rangle$. For two perfectly overlapping BECs the phase collapse is strongly suppressed, if the scattering lengths are equal. This is easily seen from the fact that, for the condensate mode of each ground level, the BEC self-interaction only depends on the total particle number: $\sum_{i=1,2} a_i^\dagger a_i^\dagger a_i a_i + 2a_1^\dagger a_2^\dagger a_1 a_2 = \hat{N}^2 - \hat{N}$, where $\hat{N} = a_1^\dagger a_1 + a_2^\dagger a_2$. Similarly, for the case of approximately equal scattering lengths the nonlinearity of the atom dynamics is reduced. In ^{87}Rb [3,4], the scattering lengths satisfy $u_1 : u_{12} : u_2 : 1.03 : 1 : 0.97$, where u_1 (u_2) denotes the intraspecies scattering length for state $|1, -1\rangle$ $(|2, 1\rangle)$ and u_{12} is the interspecies scattering length.

We write the equations of motion for the expectation values $\sigma_{ij} \equiv \langle a_i^\dagger a_j \rangle / N$, where a_i denotes the BEC annihilation operator and $\phi_i(\mathbf{r})$ is the corresponding spatial wave function. The total initial number of BEC particles is denoted by N. For simplicity, we assume approximately equal scattering lengths and approximate the equations of motion for atomic expectation values to be linear. The off-diagonal element σ_{21} describes the macroscopic coherence, or the off-diagonal long range order (ODLRO), between the two BECs. The effect of atomic interactions is treated by a phenomenological damping parameter, γ, in the equation for σ_{21}. This damping includes contributions from both the quantum effects of the BEC self-interactions and collisions between condensate and non-condensate atoms. It is assumed that these dominate over other damping mechanisms. We assume a perfect spatial overlap between the two BECs resulting in $\phi_1 = \phi_2$. The Rabi frequencies are defined by $\Omega_A \equiv 2\int d^3r \phi_3^* \phi_1 \mathbf{d}_{31} \cdot \mathbf{E}_A/\hbar$, $\Omega_B \equiv 2\int d^3r \phi_3^* \phi_2 \mathbf{d}_{32} \cdot \mathbf{E}_B/\hbar$, where $\mathbf{d}_{ij}$ denotes the electric dipole moment for the transition $i \leftrightarrow j$. In the rotating-wave approximation we obtain

$$\dot{\sigma}_{11} = \frac{\Gamma_1}{2}\sigma_{33} + \Omega_A \mathrm{Im}(\sigma_{31}), \tag{1a}$$

$$\dot{\sigma}_{22} = \frac{\Gamma_1}{2}\sigma_{33} - \Omega_B \mathrm{Im}(\sigma_{23}), \tag{1b}$$

$$\dot{\sigma}_{33} = -\Gamma_1\sigma_{33} - \Omega_A \mathrm{Im}(\sigma_{31}) + \Omega_B \mathrm{Im}(\sigma_{23}), \tag{1c}$$

$$\dot{\sigma}_{21} = i(\delta_{23} - \delta_{31} + i\gamma)\sigma_{21} + \frac{i\Omega_A}{2}\sigma_{23} - \frac{i\Omega_B}{2}\sigma_{31}, \tag{1d}$$

$$\dot{\sigma}_{23} = i(\delta_{23} + i\Gamma_2)\sigma_{23} + \frac{i\Omega_B}{2}(\sigma_{22} - \sigma_{33}) + \frac{i\Omega_A}{2}\sigma_{21}, \tag{1e}$$

$$\dot{\sigma}_{31} = -i(\delta_{31} - i\Gamma_2)\sigma_{31} + \frac{i\Omega_A}{2}(\sigma_{33} - \sigma_{11}) - \frac{i\Omega_B}{2}\sigma_{21}, \tag{1f}$$

where Im denotes the imaginary part. For simplicity, we have set Ω_A and Ω_B to be real.

In equation (1) spontaneous emission is described by the radiative decay rates Γ_1 and Γ_2. We set $2\Gamma_2 = \Gamma_1$. In general, atoms at high densities do not respond to driving light fields individually and the scattering is modified by the presence of neighboring atoms. The decay rate Γ_1 should denote, instead of the linewidth of an isolated atom, a collective linewidth [22,23] indicating a cooperative optical response. A careful analysis of light scattering from dense atomic gases would typically require a full quantum field theoretical approach [24]. In the context of the present work we only note that, if, *e.g.*, the shape of the gas is flat and the light is shone through the thin dimension, multiple scattering can be negligible and Γ_1 may be described by the linewidth of an isolated atom.

In CPT the coupling of the light fields to atoms is often described in terms of symmetric and antisymmetric coherent superpositions of the two ground states [9]. If the magnitudes of the two Rabi frequencies are equal, the superposition $|C\rangle \propto |1\rangle + \exp(i\Delta\varphi)|2\rangle$ can interact with the optical fields, where $\Delta\varphi = \arg(\Omega_B^* \Omega_A)$ is the phase difference between the Rabi frequencies. However, at two-photon resonance the transition matrix element between level $|3\rangle$ and the superposition $|NC\rangle \propto |1\rangle - \exp(i\Delta\varphi)|2\rangle$ completely vanishes. Level $|NC\rangle$ is often referred to as a non-coupled state. The cancelation of the oscillating electric dipole between $|3\rangle$ and $|NC\rangle$ may be explained in terms of a destructive interference of the transition amplitudes induced by the two driving light fields. Because an atom prepared in $|NC\rangle$ cannot escape by absorbing a laser photon, it remains "trapped" in that state. On the other hand, atoms can be optically pumped into $|NC\rangle$ *via* the spontaneous emission from $|3\rangle$, because state $|C\rangle$ is not stable against absorption. In the absence of decoherence the population becomes trapped in the non-coupled state after a few radiative lifetimes.

In the density matrix description (Eq. (1)) the electric dipole matrix elements, driven by the light fields, are described by the coherences σ_{23} and σ_{31}. In the case of thermal atoms the coherence σ_{21} is present only as a result of the two-photon light coupling between levels $|1\rangle$ and $|2\rangle$. For BECs in Zeeman sublevels $|1\rangle$ and $|2\rangle$ the coherence σ_{21} can exist as a consequence of the macroscopic quantum coherence of the BECs. As an example we consider a situation where a BEC is first prepared in level $|1\rangle$ and half of the BEC atoms are then coherently transferred to level $|2\rangle$, so that a well-established coherence between the two BECs is preserved. The relative phase between the two BECs vanishes and in equation (1) we have $\sigma_{11} = \sigma_{22} = \sigma_{21} = 1/2$. Under conditions where the phase diffusion vanishes ($\gamma = 0$), the light fields are two-photon resonant ($\delta_{31} = \delta_{23}$), and $\Delta\varphi = \pi$, this corresponds to a steady-state situation. The BECs form the non-coupled superposition state and the absorption described by $\mathrm{Im}(\sigma_{31})$ and $\mathrm{Im}(\sigma_{23})$ vanishes. This is very different from thermal atoms, in which case any coherence in the absence of the driving light fields would decay rapidly and the population trapping in $|NC\rangle$ would take several radiative lifetimes. If the BECs are initially in the non-coupled state $|NC\rangle$ and the non-condensate atoms in ground levels $|1\rangle$ and $|2\rangle$ do not exhibit macroscopic quantum coherence, the number of condensate atoms in CPT could grow as a consequence of optical pumping until the non-condensate atoms become trapped in $|NC\rangle$ as proposed by Savage *et al.* [21]. It is interesting to emphasize that the non-coupled state of the BECs is independent of the radiative linewidths.

It is easy to obtain the general steady-state solution at the two-photon resonance, $\delta_{31} = \delta_{23}$, and in the absence of phase diffusion, $\gamma = 0$, [6–9]. In the general case the Rabi frequencies Ω_A and Ω_B may be complex and we obtain for the populations $\sigma_{11} = \sigma_{22} = 1/2$ and for the coherence $\sigma_{21} = \exp[i(\Delta\varphi - \pi)]/2$ (for $|\Omega_A| = |\Omega_B|$). Independently of the initial conditions the BECs become trapped in the non-coupled superposition state $|NC\rangle$. It may be surprising that the BECs exhibit a well-defined coherence, even though their relative phase could have initially been completely undefined. The back action of different quantum measurement processes can establish the coherence for BECs [12,20,25]. However, the phase information is typically expected to be lost in a coupling to an environment if the ensemble averages of possible outcomes for measurement results are considered. In the present case the coherence of the BECs is established by the photons scattering into the non-coupled superposition state, even in the ensemble averages of all possible measurement results. The steady-state value of the phase is deterministic, instead of random, and independent of the measurement histories. As shown *via* stochastic simulations in reference [20], the detections of spontaneously scattered photons first establish a stochastically determined relative phase between the BECs. If the lasers are two-photon resonant, the value of the phase later drifts to the steady-state value determined by the phase difference of the light fields.

The state of a BEC pair is expected to be a statistical mixture due to decoherence caused by the interactions between condensate and non-condensate atoms [14,15]. In that case a relative phase between the two BECs may be prepared by measurements introducing a quantum entanglement between the BECs. The phase established in a measurement process is typically stochastic [12,25] as a result of the probabilistic outcome of a quantum measurement. A BEC pair with a controllable value of the relative phase could be prepared in CPT by choosing phase difference between the driving light fields. The coherence of the BECs in CPT indicates an entanglement between the atom numbers of the two BECs and the quantum state of the BECs has the form: $\sum_{n=1}^N c_n|n, N-n\rangle$. Recently, deterministic entanglements in Bell-type states of trapped ions have raised interest in the context of quantum computation [26].

If the phase diffusion is non-vanishing ($\gamma \neq 0$), the two-photon resonant non-coupled state with $\sigma_{11} = \sigma_{22} = 1/2$, $\sigma_{21} = \exp[i(\Delta\varphi - \pi)]/2$, and $|\Omega_A| = |\Omega_B|$, is no longer stable. In the presence of phase diffusion, $\mathrm{Im}(\sigma_{31})$ and $\mathrm{Im}(\sigma_{23})$ become non-zero and atoms start accumulating in level $|3\rangle$ due to the absorption of light.

The strength of the population trapping in the non-coupled state $|NC\rangle$ depends on the phase diffusion rate γ. The steady-state solution of equation (1) for $\gamma \neq 0$ still exhibits CPT if $|\Omega_A|$, $|\Omega_B|$, $\Gamma_1 \gg \gamma$ [8]. The atoms are trapped in $|NC\rangle$; the population in $|3\rangle$ and the absorption of light are negligible. However, if the phase diffusion is fast enough so that $\gamma \simeq \Gamma_1$, the ODLRO between the BECs vanishes and the atoms are no longer trapped in $|NC\rangle$. The steady-state values for σ_{33}, $\mathrm{Im}(\sigma_{31})$, and $\mathrm{Im}(\sigma_{23})$ are not small. The phase damping parameter γ may be determined by measuring the population in level $|3\rangle$ or the absorption of radiation.

The steady-state solution for σ_{33} has a simple form at the exact two-photon resonance ($\delta_{31} = \delta_{23}$) and with $\Omega_A = -\Omega_B$:

$$\sigma_{33} = \frac{s/2}{1 + 4\delta_{31}^2/\Gamma_1^2 + 3s/2 + s\Gamma_1/(2\gamma)}, \tag{2}$$

where $s = 2\Omega_A^2/\Gamma_1^2$ is the optical saturation parameter. The matter wave coherence between the BECs at

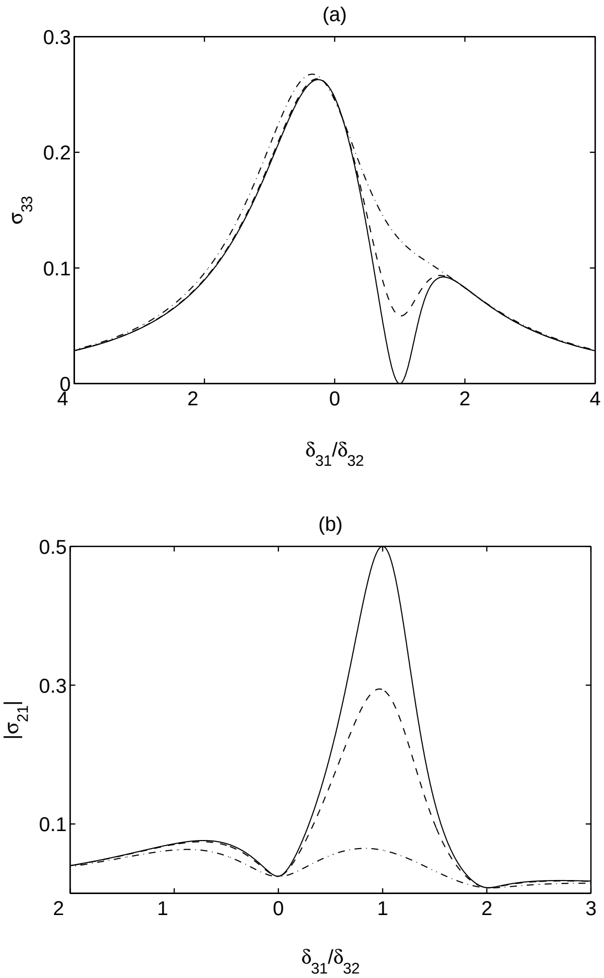

Fig. 1. A steady-state solution for (a) σ_{33} proportional to the population of the excited level as a function of the detuning δ_{31}. (b) $|\sigma_{21}|$ proportional to the macroscopic quantum coherence of the BECs (ODLRO). The solid line is the result without the phase diffusion $\gamma = 0$. The dashed line represents $\gamma = 0.1\Gamma_1$ and the dashed-dotted line represents $\gamma = \Gamma_1$.

$\delta_{31} = \delta_{23}$ is given by $\sigma_{21} = \Gamma_1\sigma_{33}/(2\gamma)$ and the absorption of light by $\mathrm{Im}(\sigma_{23}) = \mathrm{Im}(\sigma_{31}) = -\Gamma_1\sigma_{33}/(2\Omega_A)$. To leading order in the small parameter γ/Γ_1 we obtain $\sigma_{33} \simeq \gamma/\Gamma_1$, $\mathrm{Im}(\sigma_{23}) \simeq -\gamma/(2\Omega_A)$, and $\sigma_{21} \simeq 1/2 - \gamma/(s\Gamma_1)(1+4\delta_{31}^2/\Gamma_1^2+3s/2)$. For small δ_{31}^2 the BECs are in a pure state if $\gamma \ll \Gamma_1$ and $\gamma\Gamma_1 \ll \Omega_A^2$.

In Figure 1 we have plotted the steady-state solutions of (a) σ_{33} and (b) $|\sigma_{21}|$ from equation (1) for different values of γ/Γ_1 as a function of the detuning δ_{31}. We have chosen $\Omega_A = -\Omega_B = \delta_{32} = 5\Gamma_1$. The solid line is the result without the phase diffusion $\gamma = 0$. As a signature of CPT, the graph shows a strongly reduced population in the excited level $|3\rangle$ at the two-photon resonance ($\delta_{31} = \delta_{32}$). If the field $\mathbf{E}_B$ was resonant ($\delta_{32} = 0$), the curve would be symmetric around $\delta_{31} = 0$. The dashed line represents $\gamma = 0.1\Gamma_1$ and the dashed-dotted line represents $\gamma = \Gamma_1$. The accumulating population in the excited level at the two-photon resonance is clearly observed as γ increases. The phase diffusion may be detected by measuring the population in level $|3\rangle$ or the absorption of light. The absorption of light, $\mathrm{Im}(\sigma_{23})$ and $\mathrm{Im}(\sigma_{31})$, as a function of δ_{31} has the same shape as σ_{33}. The narrow resonances also appear in $\mathrm{Re}(\sigma_{23})$ and $\mathrm{Re}(\sigma_{31})$ indicating a pronounced inversion in the dispersive response [6–9]. In (b) the light fields have established the matter wave coherence, or ODLRO, between the two BECs at the two-photon resonance in the absence of phase diffusion. The reduced coherence for non-vanishing phase diffusion rates is clearly observed.

The phase diffusion rate could possibly be observed in CPT if Γ_1 was not too large. With especially weak phase diffusion rates [5] this may require a metastable excited state with the radiative linewidth of the order of a hundred Hz. Even if the radiative linewidth was very small, the number of scattered photons (approximately $\propto N\Gamma_1$) could still be large due to the Bose enhancement. We also note that the diffusion rate may significantly vary depending, *e.g.*, on the magnitudes of the scattering lengths. It is also required that the decoherence introduced by the multiple light scattering and the scattering to the non-condensate modes is slower than the phase diffusion due to the atomic interactions.

The atomic interactions may also introduce non-radiative longitudinal relaxation [8]. This damping would tend to equalize the populations of the ground levels resulting in the following additional terms to equations (1a, 1b): $\dot{\sigma}_{11} = \ldots - \gamma'(\sigma_{11} - \sigma_{22})$ and $\dot{\sigma}_{22} = \ldots - \gamma'(\sigma_{22} - \sigma_{11})$. For simplicity, we have ignored any non-radiative longitudinal relaxation. We note that this relaxation process is inconsequential at the exact two-photon resonance ($\delta_{31} = \delta_{23}$).

In conclusion, we have proposed a method of measuring the diffusion of the relative phase between two BECs in CPT. As a consequence of phase diffusion the coherence of the non-coupled superposition state is reduced and atoms absorb light. We have also shown that by using CPT the ODLRO between two BECs may be established without measurement-induced back action, even if the BECs were initially in a statistical mixture without any phase information. The value of the relative phase could be prepared deterministically, without the inherent uncertainty of the outcome of a quantum measurement.

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